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ABSTRACT

Using ordering theory, a boolean algebraic measurement model in which item response patterns are viewed as atoms in a boolean algebra with as many generators as there are items being considered, a hierarchy-generative procedure is developed. This procedure relates to the determination of prerequisite relationships between pairs of items. An experiment in which seven Piagetian tasks were administered, and the task response patterns were subjected to an ordering-theoretic analysis, is presented. It is concluded that ordering theory should be a useful tool for the educator or social scientist, and for developmental psychology and Piagetian theory and research, ordering theory should be helpful in providing information on hierarchial structures amidst behaviors. (DB)

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The Generation of Item Hierarchies by an Ordering-Theoretic Method and a Piagetian Example

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I. Introduction

Ordering theory was first introduced as a measurement model that served as an extension of Guttman's scalogram analysis (1940) and that would allow for the testing or the determination of item hierarchies (Airasian and Bart, 1971). Bart and Krus (in press) further articulated ordering theory by explicating some aspects of ordering-theoretic data analysis.

Ordering theory is a boolean algebraic measurement model in which item response patterns are viewed as atoms in a boolean algebra with as many generators as there are items being considered. The basic requirement for its usage is that one is concerned with dichotomously scored items. Also it is important that all of the subjects in a sample respond to all of the items to be analyzed.

Presently, work is occurring on the statistical underpinnings of ordering theory. It is anticipated that substantial changes in measurement and psychometrics will take place as further extensions of ordering theory. One reason is that logical relationships among items are of interest to the ordering theorist whereas correlational relationships among items are the concern of the conventional measurement theorist. Also, different data reduction procedures are used in ordering theory than are used in conventional measurement theory.

II. A Hierarchy-Generative Procedure

The procedure to be articulated relates to the determination of prerequisite relationships between pairs of items. Other ordering-theoretic methods may be developed that will be sensitive to other logical relationships. The prerequisite relationship is considered here, since that type of relationship is of primary interest to behavioral scientists - especially in their quest for causal relationships among phenomena.

An item i is a prerequisite to an item j to the extent that the (0,1) response pattern, where 0 represents the score on item i and 1 represents the score on item j, for items i and j occurs infrequently. The (0,1) response pattern is viewed as a disconfirmatory response pattern in relation to a prerequisite relationship between items in an item pair. Given that there may be error in response patterns for various reasons, tolerance levels may be set for the percentage of disconfirmatory response patterns that will be accepted before a prerequisite relationship is discarded. Thus, for a 5% tolerance level and N subjects, one would tolerate at most $(.05)\cdot N$ disconfirmatory response patterns before a prerequisite relationship between items in an item pair would be rejected. Prerequisite relationships are accepted between items in item pairs until enough disconfirmatory response patterns are recorded to exceed established tolerance levels.

A computer program has been constructed that will search out all of the prerequisite relationships between items in the item pairs for various tolerance levels and then will list the various prerequisite relationships. It is recommended that tolerance levels be small, of the order of 1%, 2%, and 5%.

III. Experimental Method

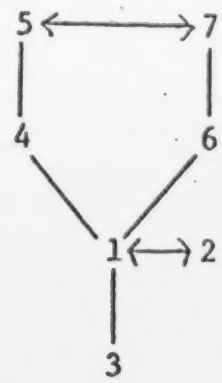
A set of seven tasks which have been articulated in The Growth of Logical Thinking from Childhood to Adolescence and The Early Growth of Logic in the Child were used. These tasks include the following: 1) Matrix task; 2) Tactile Seriation task; 3) Animal Classification task; 4) Equilibrium in the Balance Task; 5) Projection of Shadows task; 6) Oscillation of a Pendulum task; 7) Conservation of Motion in a Horizontal Plane task. These seven tasks were administered one at a time to thirty high school freshmen in individual testing situations. The subjects were in the 13-15 year age range and consisted of 16 males and 14 females. The protocols for correct task performance established by Piaget and his associates were strictly followed. The first three tasks cited measured concrete reasoning schemata and the latter four measured formal operational schemata.

IV. Results

The responses to each task were scored in a bivalent manner. The task response patterns were then subjected to an ordering-theoretic analysis with a 1% tolerance level being established. Table 1 depicts the ordering for the seven tasks. Note that the ordering is similar to a hierarchy.

Table 1

Ordering Diagram for Seven Piagetian Tasks*



Key: 1 - Matrix tests
2 - Tactile Seriation task
3 - Animal Classification task
4 - Equilibrium in the Balance task
5 - Projection of Shadows task
6 - Oscillation of a Pendulum task
7 - Conservation of Motion in a Horizontal Plane task

* 1% tolerance level is used

From Table 1 it is indicated that success on task 3 is a prerequisite to task 1 which in turn is equivalent to task 2. In other words, if there is success on task 1, matrix tests, there is success on task 2, tactile seriation task, and vice versa. Another way of articulating equivalency relationship is to say that for tasks 1 and 2 or tasks 5 and 7, all subjects either succeeded on both tasks in the pair or failed both tasks, but less than 1% of the subjects attained different scores for the tasks in each pair. Other logical relationships that may be cited for the seven tasks are that success on task 2 is a prerequisite to success on task 4 and success on task 6 is a prerequisite to success on task 7. All tasks are sufficient conditions to a given task if lines connect the tasks to the given task in a general downward direction.

One prominent finding from Table 1 is that the concrete operational schemata required by tasks 1-3 are necessary but not sufficient conditions for the formal operational schemata required by tasks 4-7. That finding is very compatible with Piagetian theory insofar as Piaget, in a host of books

(e.g., Inhelder and Piaget 1958), has contended that concrete reasoning is a precondition to formal reasoning. Another prominent finding is that the seven tasks are closely interrelated with an array of prerequisite relationships that generates a hierarchy richer than that of linear hierarchy. Within the hierarchy of the seven tasks, the two tasks (4 and 5) that assess the schema of proportionality, the two tasks (6 and 7) that assess combinatorial reasoning, and the two tasks (1 and 3) that assess classification abilities each determine a linear scale.

V. Discussion

The possible uses of ordering-theoretic methods for education and other social sciences are myriad. Wherever logical relationships between phenomena or events are of interest to a researcher, ordering theory can and should be used. Traditional psychometric theory has given to us a family of techniques that allow us to discern patterns of association, in terms of correlations. However, such techniques do not greatly help us in the determination of lines of causation which must be identified in order for the social scientific fields to develop rapidly. Ordering theory can reveal lines of implication amidst phenomena, which, in turn, can serve as a basis for hypothesizing lines of causation to be tested in experimental settings. Ordering theory should be a useful tool to the educator or social scientist.

As for developmental psychology and Piagetian theory and research, ordering theory should be helpful in providing information on hierarchial structures amidst behaviors which in turn constitute a common and useful way to conceptualize behaviors from a developmental viewpoint. In the case of this study, the hierarchy among the cognitive behaviors assessed is

very compatible with the hierarchy among cognitive behaviors which is posited by Piaget. It is anticipated that other cognitive behavior hierarchies posited by Piaget will be empirically verified using ordering-theoretic methods.

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